TOPIC 1 OVERVIEW

Circles and Ratios

How are the key concepts of *Circles and Ratios* organized?

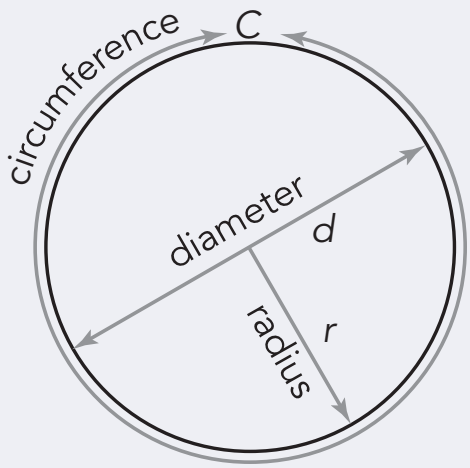
In *Circles and Ratios*, students develop formulas for the circumference and area of circles and use them to solve problems.

Students begin the topic with an introductory lesson on the problem-solving model. They will use this model throughout the course when solving problems.

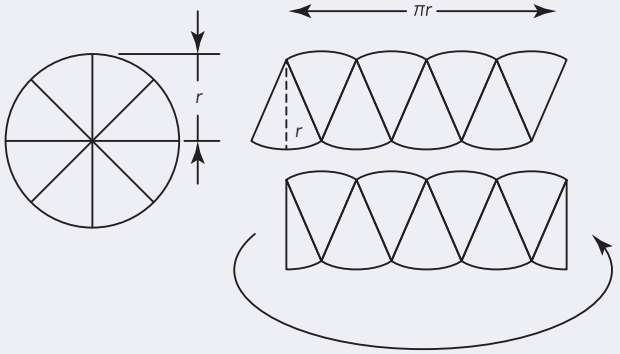
Students review the terminology of circles. They write ratios of the measures of the distance around and across different circles, noting that this ratio is constant. They learn that the irrational number π is the ratio of a circle's circumference and diameter lengths. Students use this relationship to write a formula for the circumference of a circle. They then decompose a circle and rearrange the pieces to form a familiar shape to derive the formula for a circle's area.

Math Representation

Labeled Circle



Relationship between Circumference and Area



Students use the formulas for circumference and area to solve mathematical and real-world problems.

What is the entry point for students?

Students are familiar with circles from elementary school. They have determined the perimeters of shapes and the areas of rectangles, parallelograms, and trapezoids. In Grade 6, students reasoned extensively with ratios. They used various tools to write equivalent ratios: strip diagrams, double number lines, ratio tables, and graphs. Students know how to scale ratios up and down to solve real-world and mathematical problems. To begin *Circles and Ratios*, students draw on these experiences using physical tools to investigate the constant ratio and review basic ideas of ratios and proportional relationships.

Math Representation

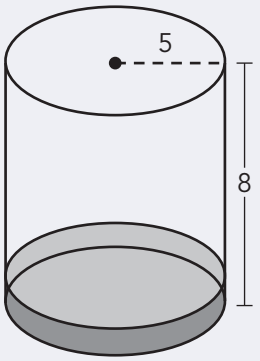
Lucas swims 3 laps in 4 minutes. You can write his swimming rate as $\frac{3 \text{ laps}}{4 \text{ minutes}}$.
You can scale up to predict how many laps Lucas could swim in 20 minutes.

$$\frac{\text{laps}}{\text{minutes}} \rightarrow \frac{3}{4} \overset{\times 5}{=} \frac{\boxed{15}}{20}$$

Lucas can probably swim 15 laps in 20 minutes.

Math Representation

Consider the cylinder. Suppose there is a circular disc of height 1 unit at the bottom of the cylinder.



The area of the top of the disc is approximately 78.5 square units. Because the height is 1 unit, each disc has a volume of approximately 78.5 cubic units.

Eight circular discs fit into the cylinder because the cylinder's height is 8 units. So, the volume of the cylinder is 628 cubic units.

The formula for the volume of a cylinder is the area of the base times the height: $V = Bh = \pi r^2 h$.

Why is *Circles and Ratios* important?

This topic provides a solid bridge between the mathematical work with ratio, equations, and area from Grade 6, with related work around proportional reasoning and the area and circumference of circles in Grade 7. This early review of and experience with ratios prepares students for the remainder of the module. Students move from concrete representations and reasoning about ratios and proportions, to more abstract and symbolic work with solving and representing proportional relationships. Pi, although not named as an irrational number in this topic, is the first irrational number students encounter. In future grades, students will use the circumference and area formulas of circles to calculate surface areas and volumes of cylinders and composite three-dimensional shapes that include circles.

How does a student demonstrate understanding?

Students will demonstrate understanding of the standards in this topic when they can:

- Describe pi (π) as the ratio of the circumference to the diameter of a circle.
- Use models to derive and explain the relationship between circumference and area of a circle.
- Justify the formulas for area and circumference of a circle and how they relate to pi (π).
- Apply the circumference and area formulas to solve mathematical and real-world problems.

NEW KEY TERMS

- congruent [congruente]
- circle [círculo]
- radius [radio]
- diameter [diámetro]
- circumference [circunferencia]
- pi [pi]
- unit rate
- composite figure [figura compuesta]

How do the activities in *Circles and Ratios* promote student expertise in the TEKS mathematical process standards?

Each topic is written with the goal of creating mathematical thinkers who are active participants in class discourse, so the TEKS mathematical process standards should be evident in all lessons. Students are expected to make sense of problems and work towards solutions, reason using concrete and abstract ideas, and communicate their thinking while providing a critical ear to the thinking of others.

To begin this topic, students examine the problem-solving model (7.1B). Students will continue to use this model throughout the course. As students continue through *Circles and Ratios* connecting mathematical ideas (7.1F) is highlighted. Composing and decomposing circles and composite figures with circular parts is a theme in this topic. Students are expected to recognize that objects and expressions can be decomposed and composed to reveal new details. For instance, cutting a circle into wedges and rearranging them to form a pseudo-parallelogram leads to a strategy for determining the area of a circle. Also, rearranging the proportion that includes the ratios of distances around and across circles yields the formula for the circumference of a circle. Students are also expected to reason about the ratio relationship, the relationship between circumference and area, and the relationships among the shapes in composite figures as they calculate areas of the figures.

How can you use cognates to support EB students?

Cognates are provided for new key terms when applicable. Encourage students to keep a bilingual math journal, recording reflections and background knowledge on new topics, in either written or verbal format, with added visuals for clarity. Incorporate journal excerpts into a shared word wall or digital bilingual glossary, with a focus on highlighting cognates.

MODULE 1, TOPIC 1 PACING GUIDE

165-Day Pacing

1 Thinking Proportionally

TOPIC 1: Circles and Ratios

1 DAY PACING = 45-MINUTE SESSION

TEKS Mathematical Process Standards: 7.1A, 7.1B, 7.1C, 7.1D, 7.1E, 7.1F, 7.1G

ELPS: 1.C, 1.E, 1.H, 2.D, 2.E, 2.I, 3.E, 4.E, 4.F, 4.G, 4.H, 5.B

Topic Pacing: 11 Days

Lesson	Lesson Title	Highlights	TEKS*	Pacing
	Introduction to the Problem-Solving Model and Learning Resources	Students reflect on learning a new skill and the variety of ways they learn. The problem-solving model, TEKS mathematical process standards, and the Academic Glossary help students complete a problem-solving activity. Students reflect on and summarize the problem-solving process. Since the intent of this lesson is to introduce the problem-solving model and review the TEKS mathematical process standards, the focus is on process not content. Students will need access to the Academic Glossary, Problem-Solving Model Graphic Organizer, Problem-Solving Questions to Ask, and TEKS mathematical process standards which are located in the Course Guide. These materials should always be available to students throughout the course. **Materials Needed:** (located in the Course Guide) Academic Glossary, Problem-Solving Model Graphic Organizer, Problem-Solving Model Questions to Ask, TEKS Mathematical Process Standards	7.6D	1
1	**Exploring the Ratio of Circle Circumference to Diameter**	Students explore the relationship between the distance around a circle and the distance across a circle. They learn the terms *circumference*, *diameter*, and *radius*. Students use hands-on tools to measure the distances and compare the ratio of the circumference to the length of the diameter. They then use a compass to create their own circles and realize that for every circle the ratio of circumference to diameter is pi. Students practice solving for the diameter or the circumference in problems. **Materials Needed:** Centimeter Rulers, String, Compasses, Calculators with a π Key, Circles (located at the end of the lesson)	7.5B 7.8C **7.9B**	2
2	**Area of Circles**	Students explore the area of a circle in terms of its circumference. They cut a circle into sectors and fit the sectors together to form a parallelogram. The parallelogram helps students see the area of a circle in relation to its circumference: $A = \left(\frac{1}{2}C\right)r$. Students derive the area for a circle and then solve problems using the formulas for the circumference and area of circles. **Materials Needed:** Scissors, Calculators with a π Key, Problem-Solving Model Graphic Organizer, Circle Area Cutouts (located at the end of the lesson)	7.4B 7.8C **7.9B**	2

*Bold TEKS = Readiness Standard

MODULE 1, TOPIC 1 PACING GUIDE

Lesson	Lesson Title	Highlights	TEKS*	Pacing
3	**Solving Area and Circumference Problems**	Students use the area of a circle formula and the circumference formula to solve for unknown measurements in problem situations. Some of the situations are problems composed of more than one figure, and some of the situations include shaded and non-shaded regions. Students then determine whether to use the circumference or area formula to solve problems involving circles. **Materials Needed:** Problem-Solving Model Graphic Organizer	**7.9B** **7.9C**	2
End of Topic Assessment				1
Learning Individually with Skills Practice *Schedule these days strategically throughout the topic to support student learning.*				3

*Bold TEKS = Readiness Standard

MODULE 1, TOPIC 1 PACING GUIDE

165-Day Pacing

1 DAY PACING = 45-MINUTE SESSION

Day 1	Day 2	Day 3	Day 4	Day 5
TEKS: 7.6D	TEKS: 7.5B, 7.8C, **7.9B**			TEKS: 7.4B, 7.8C, **7.9B**
Introduction to the Problem-Solving Model and Lesson Resources **GETTING STARTED** **ACTIVITY 1** **TALK THE TALK**	**LESSON 1** Exploring the Ratio of Circle Circumference to Diameter **GETTING STARTED** **ACTIVITY 1** **ACTIVITY 2**	**LESSON 1** continued **ACTIVITY 3** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	**LESSON 2** Area of Circles **GETTING STARTED** **ACTIVITY 1**
Day 6	**Day 7**	**Day 8**	**Day 9**	**Day 10**
		TEKS: **7.9B**, **7.9C**		
LESSON 2 continued **ACTIVITY 2** **ACTIVITY 3** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*	**LESSON 3** Solving Area and Circumference Problems **GETTING STARTED** **ACTIVITY 1** **ACTIVITY 2**	**LESSON 3** continued **ACTIVITY 3** **TALK THE TALK**	**LEARNING INDIVIDUALLY** **Skills Practice** *This is a suggested placement. Move based on student data and individual needs.*
Day 11				
END OF TOPIC ASSESSMENT				

*Bold TEKS = Readiness Standard

How can you incorporate Skills Practice with students?

There are three Learning Individually days scheduled within this topic. The placement of these days within the topic is flexible. The intent is to distribute spaced and interleaved practice throughout a topic and throughout the year. It is not necessary for students to complete all Skills Practice for the topic and different students may complete different problem sets. You should use data to strategically assign problem sets aligned to individual student needs. You should analyze student responses from the following embedded assessment opportunities to help assess individual needs: Essential Questions, Talk the Talks, Student Self-Reflections, and End of Topic Assessments. For students who are building their proficiency, you can assign problem sets to target specific skills. For students who have demonstrated proficiency, there are extension problems of varied levels of challenge.

How can you identify whether students are ready for new learning?

The Prepare section of the Lesson Assignments and the Spaced Practice set of Skills Practice can serve as diagnostic tools. Depending on available time, you can assign the Prepare section of the Lesson Assignments as homework or as a warm-up to identify students' prior knowledge for the upcoming lesson's activities. You can also use the Spaced Practice sets of Skills Practice to analyze individual students' level of proficiency on standards from previous topics.